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Turbulent energy dissipation in density jumps

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The analysis of density jumps is reconsidered and a new approximate model, based on an estimate that most of the turbulent energy produced by the jump is dissipated, is presented. The model suggests new upper bounds for the upstream Froude number, the entrainment and the downstream height. These bounds are shown to be consistent with earlier measurements.

1. Introduction

Internal hydraulic jumps in stably stratified two-layer miscible flows (also termed density jumps) are characterized by mass transfer between the layers, which can be considerable. As opposed to the case of hydraulic jumps in channel water flow, it is not possible to correlate the characteristics of the flow upstream and downstream of such jumps using the momentum and the mass balance equations alone. A closure model for the mass entrainment between the different layers is necessary to solve the problem.

The problem has attracted the attention of several workers who have focused, as we have, on the case of one moving layer bounded by a horizontal wall on one side and a layer of stagnant fluid with close density on the other side (Wilkinson and Wood 1971; Baddour and Abbink 1983; Baddour 1987 – in the latter two papers as a limiting case).

An alternative formulation has been proposed by Macagno & Macagno (1975) and is used by Findikakis & Law (1998), who use an additional equation for the energy (kinetic and potential) transfer and assume that the mean energy loss (i.e. not including turbulent energy) in the entrainment region is a predetermined fraction of the energy loss through a jump with no entrainment at the same upstream Froude number. No entrainment is assumed for the roller region. Their approach is based on a limited number of experimental data, but, for the case of no flow in the ambient layer, leads to a strong limitation on the dimensionless entrainment relative to the flow in the main layer (14% for $Fr_1 = 5$ and 23.6% for $Fr_1 = 20$), which is not consistent with existing experimental data.

Holland *et al.* (2002) proposed a closure which is based, in addition to mass and momentum conservation, on two additional assumptions.

(i) An equation is established for the total kinetic energy, including both mean flow and turbulent energy. It is further assumed that the dissipation of the turbulent energy in the vicinity of the jump is small, i.e. that the characteristic time scale of turbulence is much larger than the characteristic streamwise scale of the jump.

(ii) The stable stratification imposes an upper limit on turbulent energy, both upstream and downstream of the density jump.

As a result, bounds emerge on the amount of entrainment, as well as the Froude numbers before and after the jump. These bounds are somewhat different from those

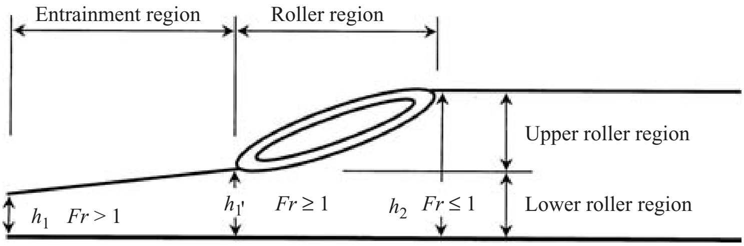

FIGURE 1. Schematic representation of a density jump.

which can be derived on the basis of the momentum and mass conservations alone as in the works of Wilkinson & Wood (1971), Baddour & Abbink (1983) and Baddour (1987), which require an additional equation for energy conservation downstream from the jump to obtain a closed set of equations.

It is the purpose of this work to examine the assumptions of Holland *et al.* and modify their analysis by modelling the energy dissipation of the turbulent energy in the region of the jump.

2. The time scale for turbulence decay

Owing to the high Reynolds number, the assumption of Holland *et al.* that the energy lost to the mean flow is converted into turbulence is well established. However, their additional assumption that it 'takes far more time (to dissipate) than the fluid spends in the neighbourhood of the jump' (p. 73) is not.

An estimate of the characteristic time scale of turbulent dissipation can be made and compared with the characteristic time of horizontal motion.

Starting from the description of the jump in stratified flows, it can be seen that the jump is usually divided (Wilkinson & Wood 1971) into two characteristic regions (figure 1).

(i) The entrainment region, in which the flow remains supercritical ($Fr_1' > 1$).

(ii) The roller region, in which the flow switches from supercritical to subcritical and in which there is no (or little) entrainment.

Denoting the height of the moving layer upstream by h_1, the height at the borderline between the entrainment and the roller region by h_1', the height at the end of the roller region by h_2 and the entrainment ratio (the additional flow rate at the end of the entrainment region divided by the initial flow rate) by $\varepsilon(\equiv(u_2h_2 - u_1h_1)/(u_1h_1))$, the mass and the momentum conservation equations can be written as:

$$u_1h_1\Delta\rho_1 = u_2h_2\Delta\rho_2, \tag{1}$$

$$u_1^2h_1 + \frac{\Delta\rho_1}{\rho}g\frac{h_1^2}{2} = u_2^2h_2 + \frac{\Delta\rho_2}{\rho}g\frac{h_2^2}{2}, \tag{2}$$

where due to the Boussinesq approximation, ρ is considered constant in the inertial terms and variable only in the gravitational ones.

As shown by Regev, Hassid & Poreh (2004), combining (1) and (2), an equation can be obtained for h_2/h_1:

$$\left(\frac{h_2}{h_1}\right)^3 - \left(\frac{h_2}{h_1}\right)(1 + 2Fr_1^2)(1+\varepsilon) + 2Fr_1^2(1+\varepsilon)^3 = 0, \tag{3}$$

where Fr_1 is the upstream Froude number, $u_1/[g(\Delta\rho_1/\rho)h_1)]^{1/2}$.

Equation (3) can only be solved for $\varepsilon < \varepsilon_{max}$, where:

$$\varepsilon_{max} = \frac{\left(1 + 2Fr_1^2\right)}{3Fr_1^{4/3}} - 1. \tag{4}$$

Two real positive solutions of (3) exist, h_1'/h_1 and h_2/h_1, the former corresponding to the end of the entrainment region and the latter to the end of the roller region. The two merge for $\varepsilon = \varepsilon_{max}$. From these values, we can estimate the mean flow energy loss through the jump at each of its two sections, which may be considered approximately equal to the extra turbulent energy ($e = \overline{u_i'^2}/2$) produced in the jump:

$$\Delta e_{tot} = 1.01\frac{q^2}{2h_1^2} + \frac{\Delta\rho_1}{\rho}gh_1 - \frac{q^2(1+\varepsilon)^2}{2h_2^2} - \frac{\Delta\rho_1}{\rho}\frac{gh_2}{(1+\varepsilon)}, \tag{5}$$

$$\Delta e_{en} = 1.01\frac{q^2}{2h_1^2} + \frac{\Delta\rho_1}{\rho}gh_1 - \frac{q^2(1+\varepsilon)^2}{2h_1'^2} - \frac{\Delta\rho_1}{\rho}\frac{gh_1'}{(1+\varepsilon)}, \tag{6}$$

$$\Delta e_{rol} = 1.01\frac{q^2}{2h_1'^2} + \frac{\Delta\rho_1}{\rho}\frac{gh_1}{(1+\varepsilon)} - \frac{q^2(1+\varepsilon)^2}{2h_1'^2} - \frac{\Delta\rho_1}{\rho}\frac{gh_1'}{(1+\varepsilon)}. \tag{7}$$

where Δe_{tot}, Δe_{en} and Δe_{rol} are the additional turbulence energy through the whole of the jump, through the entrainment region and through the roller region, respectively, and $q(\equiv uh)$ is the volume flow rate per unit width. Note that $\Delta\rho_2$ is applicable to both the end of the entrainment region and the roller region ($\Delta\rho_2 = \Delta\rho_1/(1+\varepsilon)$). The initial kinetic energy has been multiplied by 1.01, to account – approximately – for the initial turbulent energy.

As is customary in most turbulence models, the time scale of the turbulence dissipation can be defined as $\tau_{dis} = \Delta h/(2\Delta e)^{1/2}$, namely as the ratio of the turbulent length scale divided by the turbulent velocity scale, Δh being the difference in stream height before and after the jump. This can be compared to the time scale of the streamwise flow, which determines the rate of change in the horizontal mean flow direction. A reasonable estimate of such a time scale is $\tau_s = 10h_2/u_2 \approx 10h_2^2/q(1+\varepsilon)$. The factor of 10, which is usually characteristic of the ratio of the horizontal and vertical length scales, is somewhat lower than one would expect in the entrainment region and somewhat larger than one would expect in the roller region, on the basis of the angle θ between the horizontal and the upper part of the jump, approximately 8° and $\tan\theta \approx 0.14$ (Valiani 1997). It is noted that Holland *et al.* (2002, p. 82) define the jump neighbourhood as being of the order of tens of jump heights.

Three time scale ratios τ/τ_s can be derived from the above equations

(i) for the whole jump

$$\frac{\tau_{tot}}{\tau_s} = \frac{(h_2 - h_1)}{(2\Delta e_{tot})^{1/2}}\frac{0.1q(1+\varepsilon)}{h_2^2}; \tag{8}$$

(ii) for the entrainment region

$$\frac{\tau_{en}}{\tau_s} = \frac{(h_1' - h_1)}{(2\Delta e_{en})^{1/2}}\frac{0.1q(1+\varepsilon)}{h_1'^2}; \tag{9}$$

(iii) for the roller region

$$\frac{\tau_{rol}}{\tau_s} = \frac{(h_2 - h_1')}{(2\Delta e_r)^{1/2}}\frac{0.1q(1+\varepsilon)}{h_2^2}. \tag{10}$$

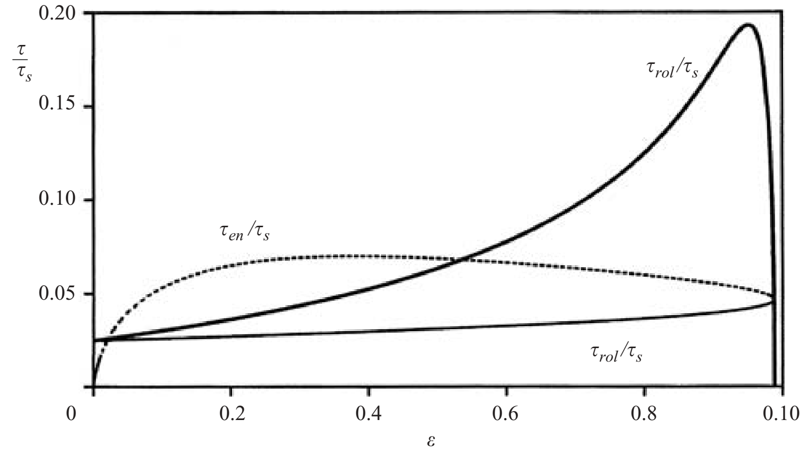

FIGURE 2. Time scale ratio *vs.* ε for upstream Froude number of 5.

The values of these three ratios for the entire range of entrainment for $Fr_1 = 5$ $(0 < \varepsilon < 0.983)$ are shown in figure 2. It is shown that all the three time scale ratios are small: they do not exceed 0.02 for the overall jump and 0.06 for the entrainment region. For the roller region larger values are found as the entrainment ratio nears its maximum value (and the roller region almost vanishes), but the time scale in this case, where the turbulent energy generated is very small, becomes of little importance. Note that since the turbulent energy estimate appears in the denominator and since the streamwise time scale is defined differently for each case, the sum of the time scales for the entrainment and the roller regions is not equal to the time scale for the whole of the jump.

We may conclude from this figure – and from similar ones for other values of the Froude number – that the time scale for turbulent dissipation is much smaller than the main horizontal motion length scale and that most of the turbulent energy produced is dissipated locally rather than in the region downstream of the jump.

3. Modified form of the model

It is proposed here to modify the model of Holland *et al.* to account for the dissipation, while keeping most of the other simplifying assumptions, namely the flat profiles of velocity and density.

An expression for the energy dissipation has been assumed by Jiang & Smith (2001), which might be valid for the case of laminar flow of a Newtonian or a non-Newtonian fluid, in the latter case the viscosity being proportional to $|\mathrm{d}U/\mathrm{d}x|$. However, we are referring to the turbulent case, which cannot, in our opinion, be treated using a generalization of the viscosity or the non-Newtonian length scale given by Jiang & Smith. The stress proportional to the horizontal positive gradient is not the main one in the case of turbulent flow ($\mathrm{d}U/\mathrm{d}y$ is much larger in the case of a jump – although not necessarily in the case of a shock); the turbulent energy production results mainly from the large stresses in the roller region.

Turbulent energy dissipation can be modelled by introducing an approximate integrated dissipation term in the equation. In the turbulent flow model of Prandtl (1945) and Wolfshtein (1967), the dissipation is modelled as $0.4e^{3/2}/L$, L being the length scale of turbulence. In the log region, L is equal to the von Kármán constant

(0.4) multiplied by the distance from the wall. Although turbulence in the density jump is different from turbulence near a wall, it can be reasonably assumed that the characteristic length scale is of the order of $(h_2 - h_1)$ with a proportionality constant similar to the von Kármán constant. Assuming that the jump has the shape of a triangular wedge, $h_2 - h_1$ high and $10(h_2 - h_1)$ long, and integrating over the jump volume, equal to $\frac{1}{2} \times 10 \times (h_2 - h_1)^2$ the dissipation per unit width D, can be found:

$$D = \tfrac{1}{2}10(h_2 - h_1)^2\left[C(e_2)^{3/2}/(h_2 - h_1)\right] = C(e_2)^{3/2}(h_2 - h_1), \tag{11a}$$

C being approximately equal to 5. A similar expression can be obtained for the dissipation in the entrainment region only, D_{en}:

$$D_{en} = \tfrac{1}{2}10(h_1' - h_1)^2\left[C(e_1')^{3/2}/(h_1' - h_1)\right] = C(e_1')^{3/2}(h'_1 - h_1), \tag{11b}$$

where e_1' and e_2 are the turbulent energy after the entrainment region and after the jump, respectively, assumed in both cases to be uniformly distributed over the upper part of the jump, between heights h_1 and h_2. In this work, only the turbulence generated because of the jump is considered, both in the dissipation and the convective outflow terms. The turbulent energy of the initial flow is neglected, on the grounds that the pre-jump turbulent energy is to a large extent produced by shear and dissipated locally and therefore has only a small contribution to the integral total (mean flow plus turbulent) energy balance.

Thus, the equation for the total energy balance before and after the jump becomes:

$$u_1 h_1\left(\frac{u_1^2}{2} + \frac{\Delta\rho_1}{\rho} g h_1\right) = u_2 h_2\left(\frac{u_2^2}{2} + \frac{\Delta\rho_2}{\rho} g h_2\right) + u_2 e_2 (h_2 - h_1) + C e_2^{3/2}(h_2 - h_1). \tag{12}$$

Note that for $C = 0$, equation (12) is practically equivalent to the one used by Holland *et al.* – with the difference that these authors add onto the left-hand side the upstream turbulent energy and onto the right-hand side the turbulent energy produced by the boundary layer in the vicinity of the wall which is unrelated to the jump – and which is assumed to be mainly locally produced and dissipated.

A non-dimensional form of (12) can be derived using continuity equation (1):

$$\left(\frac{Fr_1^2}{2} + 1\right) = \left(\frac{Fr_2^2}{2} + 1\right)\frac{h_2}{h_1} + \frac{e_2}{(\Delta\rho_2/\rho) g h_2}\left[1 + \frac{C}{Fr_2}\left(\frac{e_2}{(\Delta\rho_2/\rho) g h_2}\right)^{1/2}\right]\left(\frac{h_2}{h_1} - 1\right) \tag{13}$$

with

$$Fr_2 = Fr_1\left[\frac{h_2}{h_1}(1 + \varepsilon)\right]^{3/2}. \tag{14}$$

An equation similar to equation (13) can be derived for the end of the entrainment region, with h_1' in place of h_2 with the downstream Froude number being larger than one (i.e. supercritical). The turbulent energy downstream of the jump e_2 can be calculated from (12) or (13) and (3) as a function of the upstream Froude number Fr_1 and the dimensionless entrainment ε.

4. Results

The dependence of the mean turbulent energy on the non-dimensional entrainment at the end of the jump, for $C = 5$ and five values of Fr_1 is shown in figure 3 (turbulent energy non-dimensionalized using $\Delta\rho_2$ and h_2 or h_1'). In figure 4, the turbulent energy

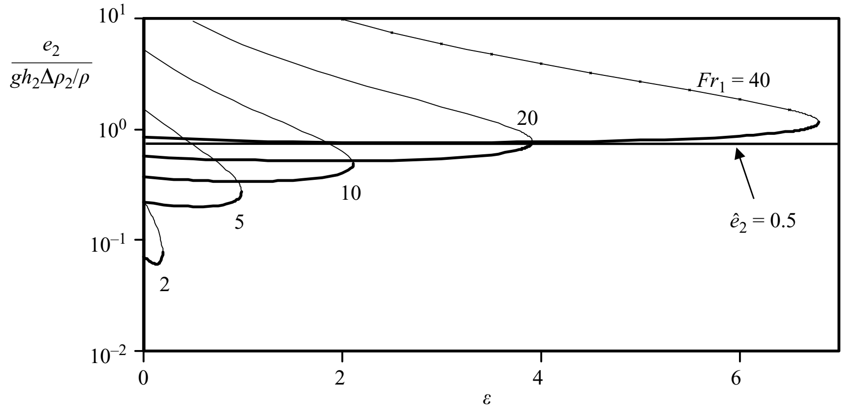

FIGURE 3. Dependence of turbulent energy non-dimensionalized using $\Delta\rho_2(\Delta\rho_1')$ and h_2 (h_1') on dimensionless entrainment ε after a density jump for different values of Fr_1 and for $C = 5$: Bold lines for $Fr_2 < 1$ (entrainment and roller region). Regular lines for $Fr_1' > 1$ (entrainment region only). The line $\hat{e}_2 = 0.5$, is for non-dimensionalized turbulent energy equal to 0.5, ($\hat{e}_2 = e_2/(\Delta\rho_2/\rho g h_2)^{1/2}$).

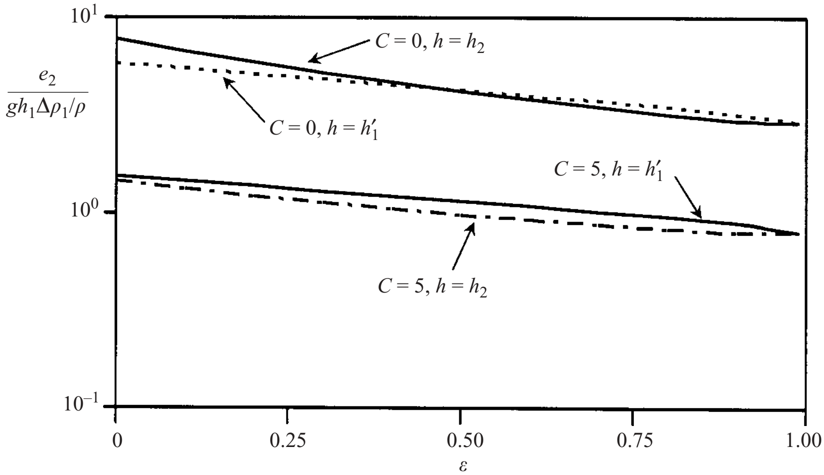

FIGURE 4. Dependence of turbulent energy non-dimensionalized using $\Delta\rho_1/\rho$ and h_1 on dimensionless entrainment ε after density jump for $Fr_1 = 5$. $h = h_1'$: entrainment region only ($Fr_1' > 1$). $h = h_2$: entrainment and roller region ($Fr_2 < 1$).

non-dimensionalized using $\Delta\rho_1$ and h_1 is shown for $Fr_1 = 5$ and two values of C, $C = 0$ (corresponding to a slight modification of the Holland *et al.* model) and $C = 5$.

In figure 3, for the jump which includes both the entrainment and the roller region, the non-dimensional turbulent energy is shown to reach a minimum at approximately 60–75% of the maximum entrainment, whereas in figure 4 the minimum is reached close to the maximum possible entrainment. If a jump over only the entrainment region is considered (i.e. $Fr_1' > 1$), the turbulent energy decreases monotonically to its minimum value, $Fr_1' = 1$, where h_2 and h_1' merge.

Note that the value of e_1' (for the entrainment region) is shown to be larger than that of e_2 (for both regions) – but there is no contradiction in that, the value of

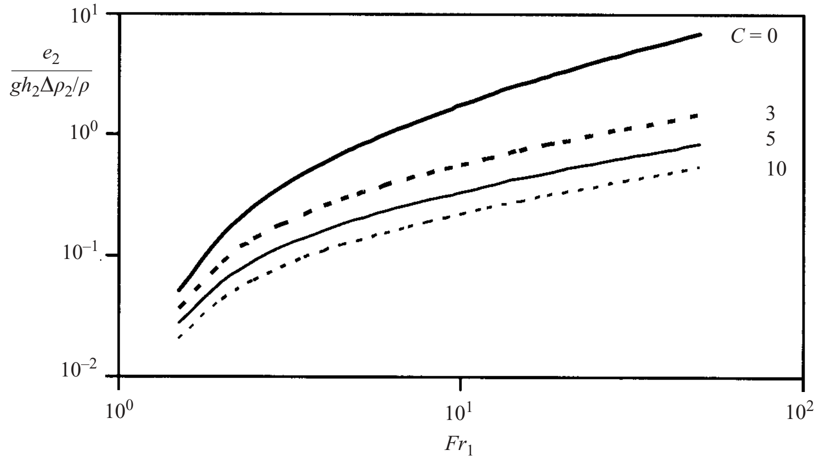

FIGURE 5. Dependence of minimum value of turbulent energy (non-dimensionalized using $\Delta\rho_2/\rho$ and h_2) on Fr_1.

turbulent energy integrated over the jump volume is smaller when one considers the entrainment region only.

The values of the non-dimensional turbulent energy are much smaller for $C=5$ – of the order of 0.2 – as compared to 0.8 for $C=0$. This indeed shows an order of magnitude difference which is expected to have a large influence on the other conclusions of the work of Holland *et al.*

In figure 5, the dependence of Fr_1 on the minimum value of the turbulent energy, non-dimensionalized using $\Delta\rho_2$ and h_2 is shown for different values of C.

5. The limitation on the value of turbulent energy and the full closure

Holland *et al.* suggest an additional limitation on the turbulent energy – stemming from momentum considerations on the w^2 (vertical) component of turbulent energy and a further assumption for the ratio $w^2/2e$ ($\approx 1/2$ for two-dimensional turbulence and $\approx 1/3$ for three-dimensional):

$$e_2 \leqslant \frac{d}{4}\frac{\Delta\rho_2}{\rho}gh_2 \tag{15}$$

with $d=2$ for two-dimensional and $d=3$ for three. Inequality (15) in turn results on limits on the values of the Froude numbers and on the other parameters characterizing the flow. Note that if it is applicable to the total turbulent energy, it also obviously applicable to e_2 as defined in this work. Note also that unlike the work of Holland *et al.*, no consideration to the similar limits upstream of the jump is given, since these limits were derived from gravitational flow considerations and do not apply to the flow upstream of the jump, which is shear dominated. This modification of the Holland *et al.* model, however, has only a secondary effect on the results.

By converting inequality (15) into an equation, namely taking the upper limit for the turbulent energy, Holland *et al.* obtain a closure for the equation describing the turbulent jump for what they call a 'strong' jump. The main result of these assumptions is that for the two-dimensional case, there can be no solution for a $Fr_1 \leqslant 1.9$ or $Fr_1 \geqslant 3.6$, whereas for the three-dimensional case, the corresponding values are $Fr_1 \leqslant 2.1$ and $Fr_1 \geqslant 4.7$ (a transfer to a subcritical flow after the jump is possible

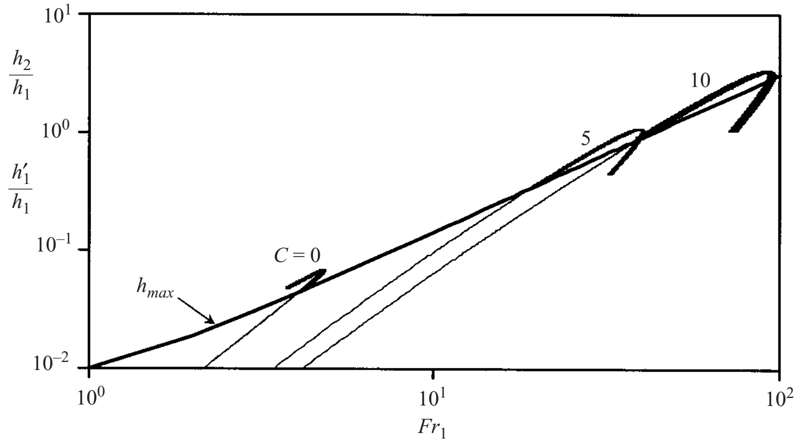

FIGURE 6. Dependence of h_2/h_1 (bold lines) and h'_1/h_1 (regular lines) on the upstream Froude number for the 'strong' jump case for $C = 0$, $C = 5$ and $C = 10$ (three-dimensional case). (Note: h_{max} is not the maximum value of h_2 (or h'_1), but the value of h_2 (and h'_1) for maximum entrainment).

only for $3 \leqslant Fr_1 \leqslant 3.6$ for the two-dimensional case and for $3.7 \leqslant Fr_1 \leqslant 4.7$ for three). Note that these values are very close to those of Holland *et al.* i.e. $1.9 \leqslant Fr_1 \leqslant 3.6$ for two-dimensional and $2.1 \leqslant Fr_1 \leqslant 4.7$ for three, in spite of the slightly different formulation which ignores the influence of upstream turbulence.

From figure 5, it can be seen that, for $C = 5$, the value of Froude number Fr_1 for which $\Delta e_2/[(\Delta\rho_2)/\rho g h_2]$ is equal to 1/2 is 18.9 for two dimensions (39.6 for three) – appreciably higher than the corresponding upper limits deduced by Holland *et al.* for $C = 0$. The corresponding lower limits for Fr_1 can be derived by considering the value of turbulent energy for $\varepsilon = 0$ and no transfer to sub-critical flow and these are 2.9 (for two dimensions) and 3.5 (for three). The latter values do not necessarily imply that a density jump is not possible, but rather that it cannot be a 'strong' one, i.e. with maximum possible turbulent energy, using the terminology of Holland *et al.*

In figure 6, the calculated dependence of h_2/h_1 and h'_1/h_1 for $C = 0$, $C = 5$ and $C = 10$ on the value of the upstream Froude number is shown for the three-dimensional 'strong jump' case. A similar picture can be obtained for the two-dimensional case. It is seen that the value of h_2/h_1 can reach much larger values of Fr_1 than in the case of $C = 0$. For $C = 5$, we can see that $3.5 < Fr_1 < 39.6$ for three dimensions ($2.9 < Fr_1 < 18.9$ for two dimensions). The values of h_2/h_1 reached are also much higher, of the order of 107 (compared to 6.7 in the case of the $C = 0$) for the three-dimensional case and 40.7 for the two-dimensional case (compared to 4.7 for $C = 0$).

In figure 7, the dimensionless entrainment is shown as a function of the upstream Froude number for the strong jump case. For the small Froude numbers, a strong jump (for which Holland *et al.* use the term 'full closure' as well) in the sense of inequality (15) becoming an equation is not possible. As a result of the maximum turbulent energy limitation, for higher numbers, the entrainment is much smaller than the maximum consistent with momentum conservation (ε_{max}). At the end of the curves, the entrainment is zero.

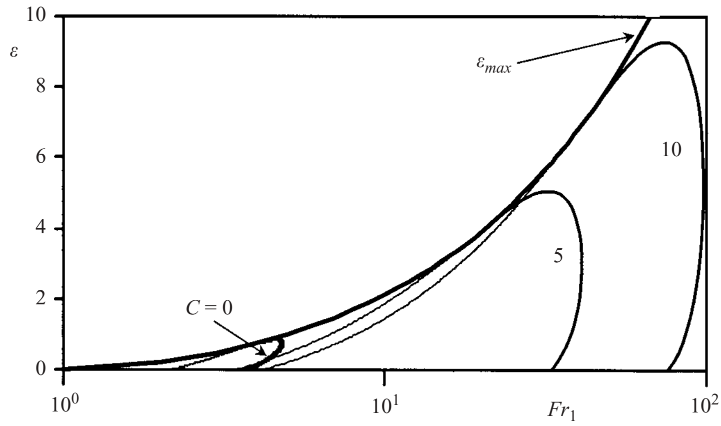

FIGURE 7. Non-dimensional entrainment for strong jump as a function of Fr_1, for $C = 0$, $C = 5$ and $C = 10$, compared to maximum entrainment compatible with conservation of momentum ε_{max}. Bold lines for jump with entrainment and roller regions ($Fr_2 < 1$), regular lines for jump with entrainment region only ($Fr_1' > 1$).

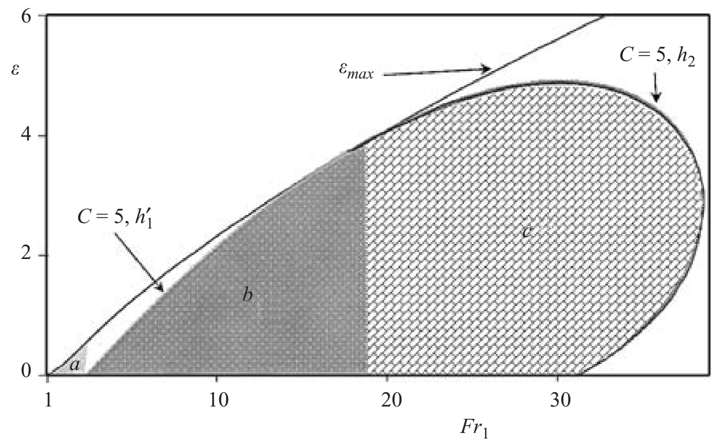

FIGURE 8. Non-dimensional entrainment for strong jump as a function of Fr_1, for $C = 5$, compared to maximum entrainment compatible with conservation of momentum ε_{max}.

In figure 9, the value of the Froude number downstream of a 'strong' jump is shown for $C = 0$, $C = 5$ and $C = 10$. It is seen that the downstream Froude number is larger than one (Fr'_1) for a part of the range and lower than one for the rest (Fr_2). The general shape of all the curves is similar, including a part in which two values of the downstream Froude number are possible, but again the upper bound of Froude number is much higher for $C = 10$.

In all cases, three different regions can be distinguished, understood best by referring to figure 8 in conjunction with figure 3.

(a) For relatively small upstream Froude numbers (for example, $Fr_1 = 2$ in figure 3), the maximum value of the turbulent energy consistent with inequality (15) cannot be achieved, and the inequality is satisfied for all values of the entrainment compatible with the momentum equation.

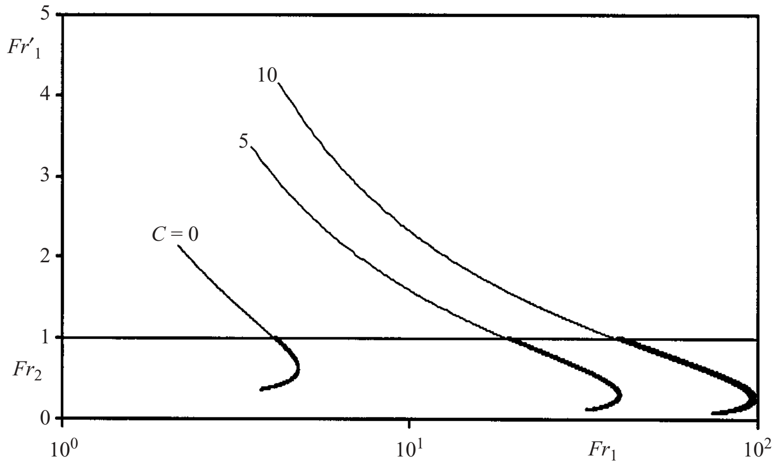

FIGURE 9. Fr_2 and Fr'_1 *vs.* Fr_1 for strong jump, for $C=0$, $C=5$ and $C=10$. Bold lines for strong jump with entrainment and roller region (Fr_2), regular lines for strong jump with entrainment region only (Fr'_1).

(*b*) Beyond the Froude number of 3.5 (2.9 for two dimensions) a strong jump, as defined by the maximum value of inequality (15), is not always possible for a jump containing a roller region (subcritical downstream). However, it is possible for a jump containing only an entrainment region (supercritical downstream). This is the case of $Fr_1=5$ and $Fr_1=10$ in figure 3. Equation (13) limits this jump between the ε_{max} curve and the maximum entrainment compatible with the inequality (15) curve. The limitation on the turbulent energy at the edge of the entrainment region is not valid when the entrainment region is followed by a roller region – since in that case kinetic energy is violently transformed into potential energy.

(*c*) The upper limit of region (*b*) is the value of Froude number for which the downstream Froude number is equal to 1 and a solution of (13) compatible with a subcritical Froude number less than one appears (for $C=5$, $Fr_1=19$ – $Fr_1=10$ for the two-dimensional case). Beyond that, (15) imposes a maximum value of entrainment smaller than that imposed by the momentum equation alone, This is exemplified by $Fr_1=20$ in figure 3. The maximum value of the Froude number, beyond which (15) admits no solution, is 39.6 (18.9 for the two-dimensional case). In part of region (*c*), we can see in figure 8 (and also in figures 6–9) that there is an upper and a lower limit to the possible entrainment.

6. Discussion and conclusions

If dissipation in the jump region is accounted for (albeit in a crude manner – as is consistent with one-dimensional analysis), the results of the model of Holland *et al.* are fundamentally affected. The range of the Froude number and the other parameters for which jumps can occur is extended and accordingly, much larger post-jump heights are possible.

Wilkinson (1970) and Wilkinson & Wood (1971) report measured density jumps for upstream Froude numbers of 16.5 and 10.5 – i.e. much higher than those suggested by Holland *et al.* Baddour & Abbink (1983) also report inlet Froude numbers as high as 21 (although they consider a Froude number of 10.8 as critically unstable), whereas Baddour (1987) also reports measured Froude numbers of 9.7. These Froude numbers are consistent with those calculated with the modified model.

The dimensionless entrainment ε in an actual flow is not determined by the model, which gives only an alternative to ε_{max} as calculated from momentum conservation considerations: it is seen in figures 4–8 that turbulent energy conservation results in a smaller value of ε_{max} and in a restriction in the value of the Froude number. The dimensionless entrainment ε can have smaller values, which are determined by downstream conditions like a weir, an obstruction or a constriction.

The limiting values of the different parameters are derived as a result of the limitation of the turbulent energy in the downstream section (in this work we do not consider the limitation on the turbulent energy applicable to the upstream section, in which the turbulent energy is scaled with the mean kinetic energy based on the mean velocity rather than the gravitational potential energy). For small values of the Froude number, there is a region where the limitation on the turbulent energy does not limit the maximum. If the jump is considered 'strong', the limiting value of the turbulent energy gives an additional equation required for a full closure. For small values of the Froude number, such a closure is not possible and in that case the amount of turbulent energy set free is not enough to reach the maximum value given.

As in all one-dimensional models, the results should be treated with caution, as many approximations are necessary to obtain results. In particular, the flat profiles of density, velocity and turbulent energy are not necessarily correct. The modelling of the dissipation is admittedly crude – and the constant chosen is approximate, based on what is customary for turbulent energy models and assuming a horizontal length scale equal to 10 times the jump height – and with no consideration for the variation of turbulent energy inside the jump. One might argue that using the turbulent energy downstream of the jump to model dissipation also introduces some further error, but there is no better alternative if one wants to keep the basic features of the one-dimensional model.

In spite of those limitations, however, it is thought that the modification of the theory of Holland *et al.* provides a better understanding of the phenomenon of density jumps and depicts the decisive role of turbulent energy dissipation on its characteristics – unfortunately at the expense of adversely affecting the beauty of the expressions derived by these authors: turbulent energy is simply not conserved through the jump and its dissipation cannot be ignored.

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